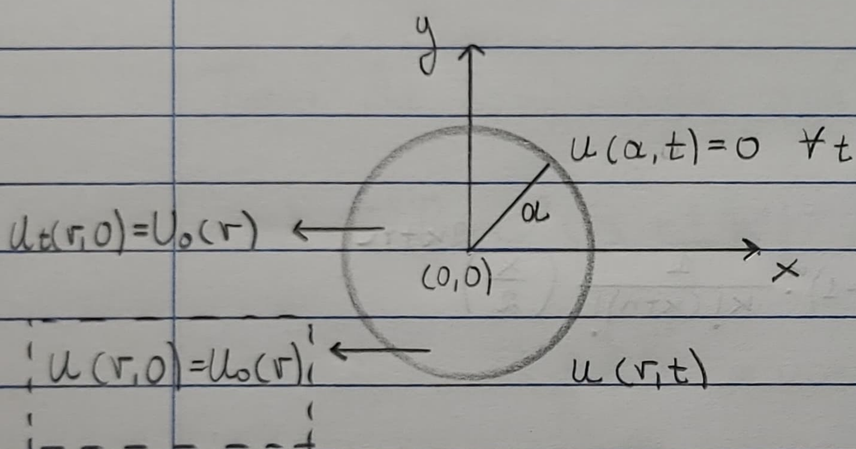
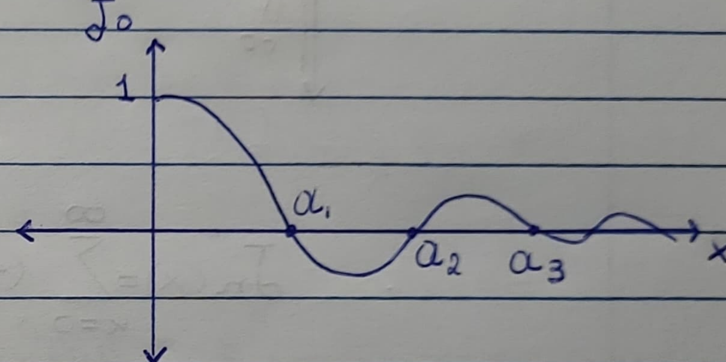
05/04/23

$$u(r,t)=\sum_{n=1}^{\infty} J_0(\lambda_n r)\left[A_n\cos(\lambda_n t)+B_n\sin(\lambda_n t)\right]$$

όπου $\lambda_n=\dfrac{\alpha_n}{\alpha}$





$$\int_0^{\alpha} J_0(\lambda_j x)\,J_0(\lambda_k x)\,x\,dx=0 \quad (\text{αν } k\neq j)$$

$$(\text{αν } k=j) \;=\; \frac{\alpha^2}{2}J_1^2(\alpha_j)$$

Οπότε,

$$u(r,0)=\sum_{n=1}^{\infty} A_n J_0(\lambda_n r)=u_0(r)$$

$$\Rightarrow \int_0^{\alpha}\sum_{n=1}^{\infty} A_n J_0(\lambda_n r)\,J_0(\lambda_m r)\,r\,dr=\int_0^{\alpha} u_0(r)\,J_0(\lambda_m r)\,r\,dr$$

$$\Rightarrow A_n \frac{a^2}{2} J_1^2(\alpha_n) = \int_0^a u_0(r) J_0(\lambda_n r)\, r\, dr$$

$$\boxed{A_n = \frac{2}{a^2 J_0^2(\alpha_n)} \int_0^a u_0(r) J_0(\lambda_n r)\, r\, dr}$$

$$u_t(r,t) = \sum_{n=1}^{\infty} J_0(\lambda_n r)\left[-A_n \lambda_n \sin(\lambda_n t) + B_n \lambda_n \cos(\lambda_n t)\right]$$

$$u_t(r,0) = \sum_{n=1}^{\infty} J_0(\lambda_n r) B_n \lambda_n$$

$$\Rightarrow \cdots \Rightarrow \boxed{B_n = \frac{2}{a^2 J_0^2(\alpha_n)\lambda_n} \int_0^a U_0(r) J_0(\lambda_n r)\, r\, dr}$$