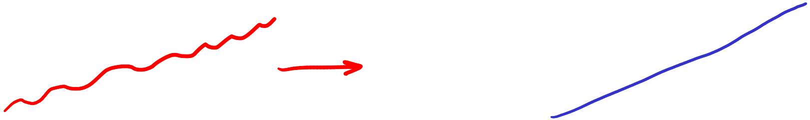


MEM-205 Περιγραφική Στατιστική

Τμήμα Μαθηματικών και Εφ. Μαθηματικών, Πανεπιστήμιο Κρήτης

Κώστας Σμαραγδάκης (kesmarag@pm.me)

03-04-2023

$Y_t \xrightarrow{\text{filter}} Y_t^*$

$Y_t = T_t + S_t + R_t \quad + \quad S_t$ p-periodic

- Έστω S_t είναι p-periodic

$$S_t = S_{t+p}, \quad t = 1, \ldots, N - p$$

- Εάν εφαρμόσουμε τον απλό κινητό μέσο p τάξης

$$S_t^* = S \quad \forall t$$

$$Y_t = \overbrace{T_t + S}^{T_t'} + \overbrace{S_t - S}^{S_t'} + R_t = T_t' + S_t' + R_t$$

- Υποθέτουμε ότι $S_t^* = 0$, ενσωματόνοντας το S στη μακροχρόνια τάση

$$T_t' = T_t + S$$

- Για ευκολία από εδώ και πέρα θα ενοούμε ως T_t το T_t'

$$S_t = [1, 0, 2, 1, 0, 2, 1, 0, 2, 1, 0, 2] \to [\sqcup, 1, 1, 1, \ldots, 1, \sqcup]$$

-3 θέσεις $\quad +3$ θέσεις

$2 = s+1 \qquad 11 = 12-s$

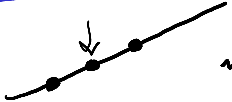
$p = 3 = 2s+1 \Rightarrow s = 1$

$[1/3, 1/3, 1/3]$

$[1/4, 1/2, 1/4]$

$[3/4, 1 + 1/4, 1/2 + 1/2]$

$Y_t = T_t + S_t + R_t$, S_t: p-periodic και $S_t^* = 0$



$T_t^* \approx T_t$

$R_t^* \approx 0$

$$Y_t^* = T_t^* + 0 + R_t^* \approx T_t$$

- Ορίζουμε τη χρονολογική σειρά με τις διαφορές $Y_t \to Y_t^*, \quad t = s+1, \dots, n-s$

$$D_t = Y_t - Y_t^* \sim S_t + R_t$$

$Y_t^* \sim T_t \qquad Y_t = T_t + S_t + R_t \Rightarrow D_t \sim S_t + R_t, \quad t = s+1, \dots, n-s$

- Ορίζουμε τα $\bar{D}_t$

$$\bar{D}_t = \frac{1}{n_t} \sum_{j=0}^{n_t - 1} D_t, \quad t = 1, \dots, p$$

- Προσεγγίζουμε τα S_t με τα $\hat{S}_t$

$$\hat{S}_t = \bar{D}_t - \frac{1}{p} \sum_{j=1}^{p} \bar{D}_j \sim S_t, \quad t = 1, \dots, p$$

- Επεκτήνουμε σε όλο το μήκος της χρονολογικής σειράς

$$\hat{S}_{t+jp} = \hat{S}_t, \quad j = 1, 2, \dots, J_t, \; t = 1, \dots, p$$

$y_t \to D_t, \quad t = s+1, \dots, n-s$ $\qquad p = 2s+1$

$D_t : \underbrace{\sqcup, \sqcup, \dots, \sqcup}_{s}, D_{s+1}, \dots, D_{n-s}, \underbrace{\sqcup, \sqcup, \dots, \sqcup}_{s}$

$S_t \quad S_1, S_2, \dots, S_s, S_{s+1}, \dots, S_p, S_1, S_2,$

για $p=3$ $\qquad D_t \approx S_t + R_t \to \bar{D}_t$ είναι προσεγγίσεις του S_t, $t=1,\dots,p$

$$\begin{array}{cccccccccccc} \sqcup, & D_2, & D_3, & D_4, & D_5, & D_6, & D_7, & D_8, & D_9, & D_{10}, & D_{11}, & \sqcup \\ S_1 & S_2 & S_3 & S_1 & S_2 & S_3 & S_1 & S_2 & S_3 & S_1 & S_2 & S_3 \end{array}$$

$\bar{D}_t, \; t=1,\dots,p.$

$$\bar{D}_1 = \frac{1}{3}\left[D_4 + D_7 + D_{10}\right]$$

$$\bar{D}_2 = \frac{1}{4}\left[D_2 + D_5 + D_8 + D_{11}\right]$$

$$\bar{D}_3 = \frac{1}{3}\left[D_3 + D_6 + D_9\right]$$

$$Y_t = \underset{\text{μεταβάλλεται αργά}}{T_t} + \underbrace{S_t + R_t}$$

1^o βήμα: $Y_t \rightarrow Y_t^*$ (προσέγγιση του T_t)

2^o βήμα: $D_t = Y_t - Y_t^*$ (προσέγγιση του $S_t + R_t$) μεταβάλλεται αργά

3^o βήμα: $\bar{D}_t = \frac{1}{\cdot} \sum D_t$ (προσέγγιση του S_t), $t = 1, \ldots, p$

4^o βήμα: $\boxed{S_t^* = S = 0}$ $\quad \dfrac{\bar{D}_1 + \bar{D}_2 + \bar{D}_3}{3}$

$$\hat{S}_t = \bar{D}_t - \frac{\bar{D}_1 + \bar{D}_2 + \bar{D}_3}{3} \approx S_t \quad , t = 1, \ldots, p$$

$$\frac{\hat{S}_1 + \hat{S}_2 + \hat{S}_3}{3} = 0$$

$$\hat{S}_t = [\hat{S}_1, \hat{S}_2, \hat{S}_3, \hat{S}_1, \hat{S}_2, \ldots\ldots,] \quad \forall t = 1, \ldots, n$$

$$Y_t - \hat{S}_t \sim Y_t - S_t = T_t + R_t \quad , \forall t$$

Απαλοιφή της εποχικής συνιστώσας

$$\mathrm{Y_t - \hat{S}_t \sim Y_t - S_t = T_t + R_t, \quad t = 1, \ldots, N}$$

Παράδειγμα

$+S$

$$T_t = [10,\ 15,\ 22,\ 24,\ 33,\ 36,\ 40,\ 50,\ 55,\ 55,\ 58,\ 60]^T$$

$S = 36/3 = 12$

$-S$

$$S_t = [10,\ 6,\ 20,\ 10,\ 6,\ 20,\ 10,\ 6,\ 20,\ 10,\ 6,\ 20]^T$$

$$R_t = [-1,\ -2,\ 1,\ 1,\ -1,\ 2,\ 0,\ 1,\ -1,\ 2,\ -2,\ 0]^T$$

$$Y_t = [19,\ 19,\ 43,\ 35,\ 38,\ 58,\ 50,\ 57,\ 74,\ 67,\ 62,\ 80]^T$$

Παράδειγμα

$$T_t = [22,\ 27,\ 34,\ 36,\ 45,\ 48,\ 52,\ 62,\ 67,\ 67,\ 70,\ 72]^T$$

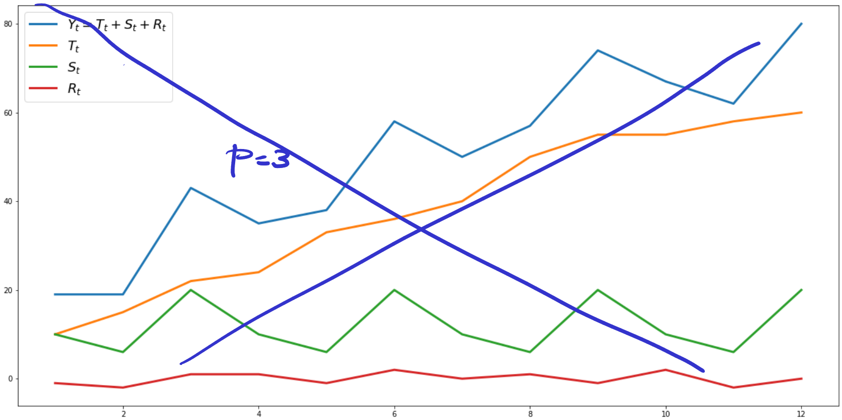
$$S_t = [-2,\ -6,\ 8,\ -2,\ -6,\ 8,\ -2,\ -6,\ 8,\ -2,\ -6,\ 8]^T$$

$$R_t = [-1,\ -2,\ 1,\ 1,\ -1,\ 2,\ 0,\ 1,\ -1,\ 2,\ -2,\ 0]^T$$

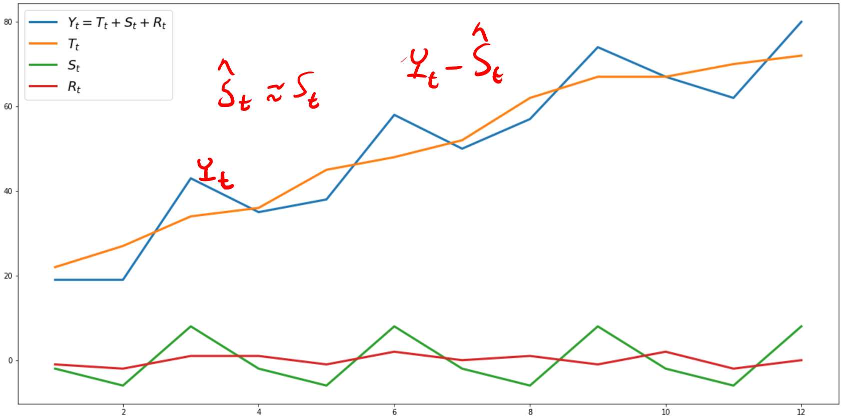
$$Y_t = [19,\ 19,\ 43,\ 35,\ 38,\ 58,\ 50,\ 57,\ 74,\ 67,\ 62,\ 80]^T$$

$p=3$

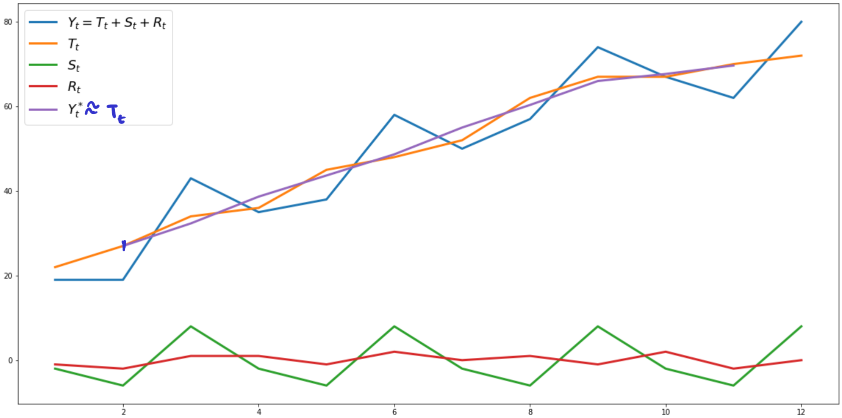
Παράδειγμα



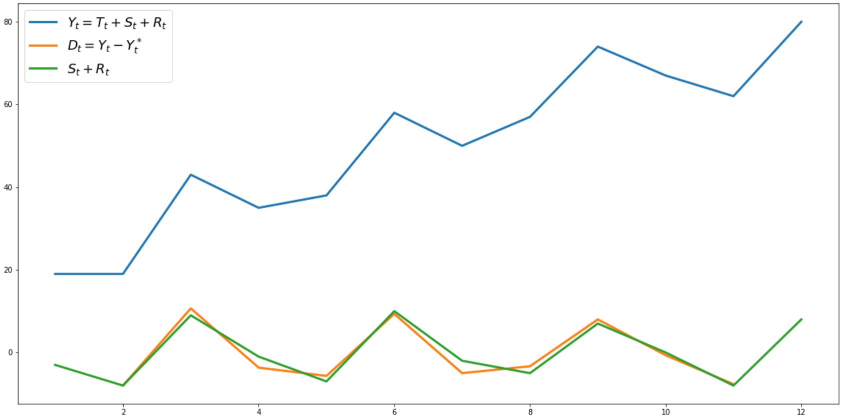
Παράδειγμα


$Y_t = T_t + S_t + R_t$
T_t
S_t
R_t
$\hat{S}_t \approx S_t$
$Y_t - \hat{S}_t$
Y_t
80
60
40
20
0
2
4
6
8
10
12

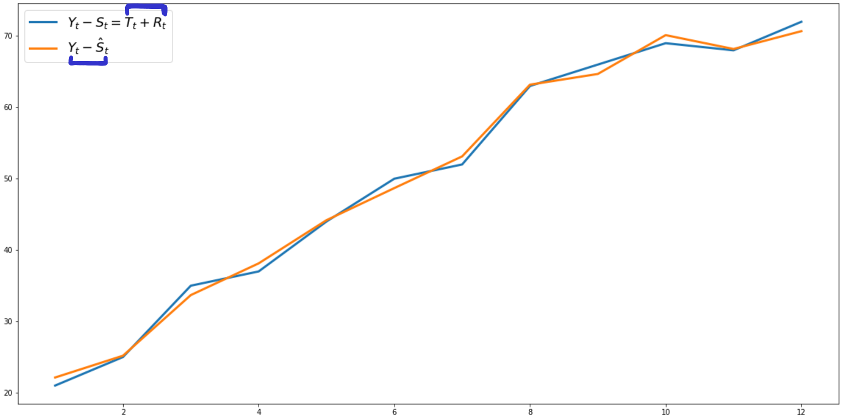
Παράδειγμα



Παράδειγμα



Παράδειγμα



Παράδειγμα

$P=4=2\cdot s \Rightarrow s=2$

$\left[\frac{1}{4s}, \frac{1}{2s}, \ldots, \frac{1}{2s}, \frac{1}{4s}\right]$ with $\underbrace{\frac{1}{2s}, \ldots, \frac{1}{2s}}_{2s-1}$

$\left[\frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8}\right]$

$$\mathrm{T_t} = [17,\ 22,\ 29,\ 31,\ 40,\ 43,\ 47,\ 57,\ 62,\ 62,\ 65,\ 67]^{\mathrm{T}}$$

$3\frac{1}{8} - 3\frac{1}{4} \cancel{-2\frac{1}{4}} \cancel{+2\frac{1}{4}} + 3\frac{1}{8} = 0$

$$\mathrm{S_t} = [3,\ -3,\ -2,\ 2,\ 3,\ -3,\ -2,\ 2,\ 3,\ -3,\ -2,\ 2]^{\mathrm{T}}$$

$-3\frac{1}{8} \cancel{-2\frac{1}{4}} \cancel{+2\frac{1}{4}} + 3\frac{1}{4} - 3\frac{1}{8} = 0$

$$\mathrm{R_t} = [-1,\ -2,\ 1,\ 1,\ -1,\ 2,\ 0,\ 1,\ -1,\ 2,\ -2,\ 0]^{\mathrm{T}}$$

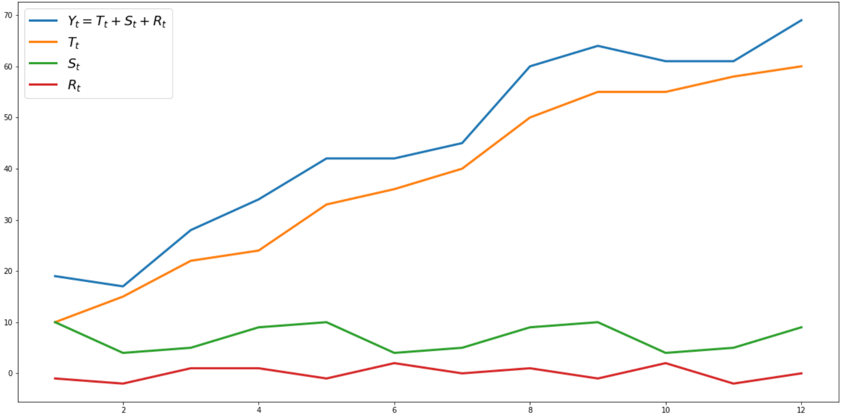
$$\checkmark\ \mathrm{Y_t} = [19,\ 19,\ 43,\ 35,\ 38,\ 58,\ 50,\ 57,\ 74,\ 67,\ 62,\ 80]^{\mathrm{T}} \qquad P=4$$

$Y_t^* = [\sqcup, \sqcup, \_\_ \ \ldots \ \_\_, \sqcup, \sqcup]$

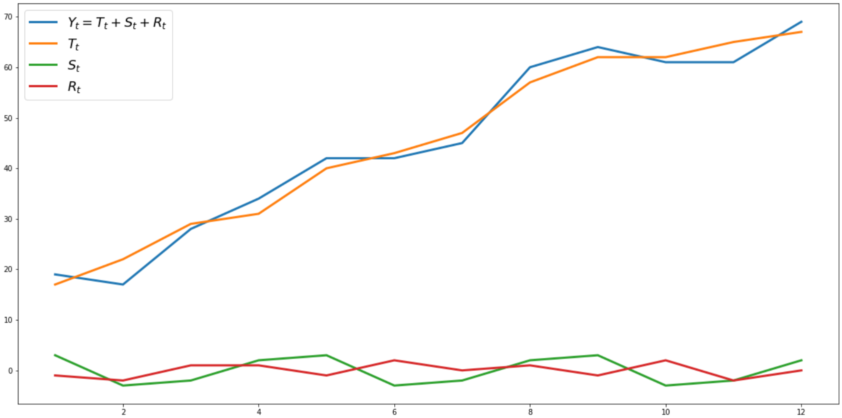
$\frac{1}{8}19 + \frac{1}{4}19 + \frac{1}{4}43 + \frac{1}{4}35 + \frac{1}{8}38$

$\frac{1}{8}57 + \frac{1}{4}74 + \frac{1}{4}67 + \frac{1}{4}62 + \frac{1}{8}80$

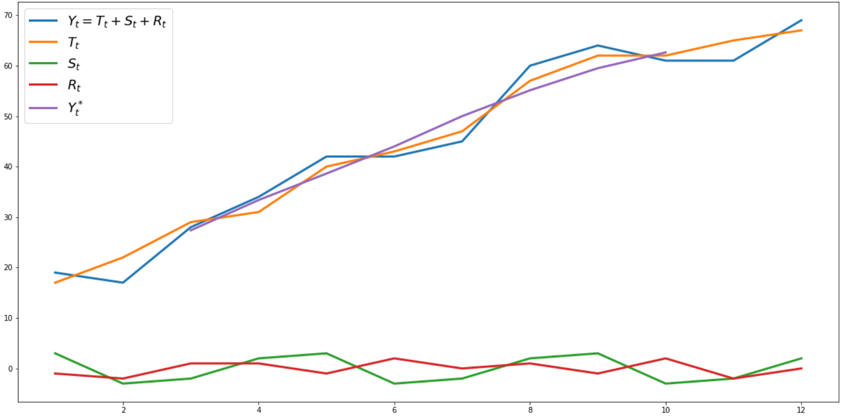
Παράδειγμα



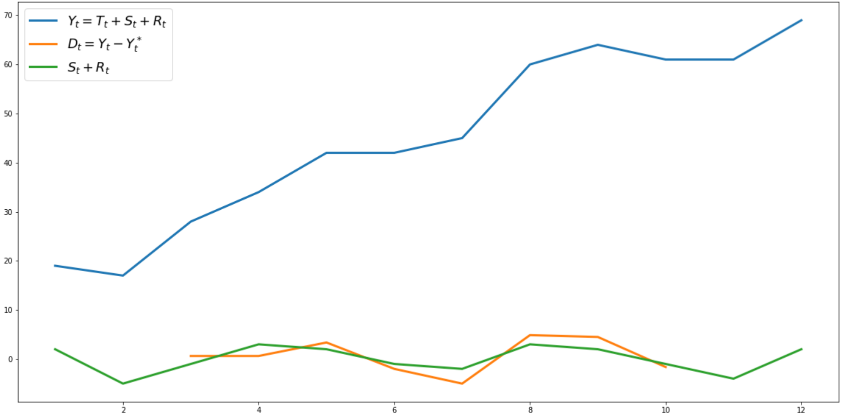
Παράδειγμα



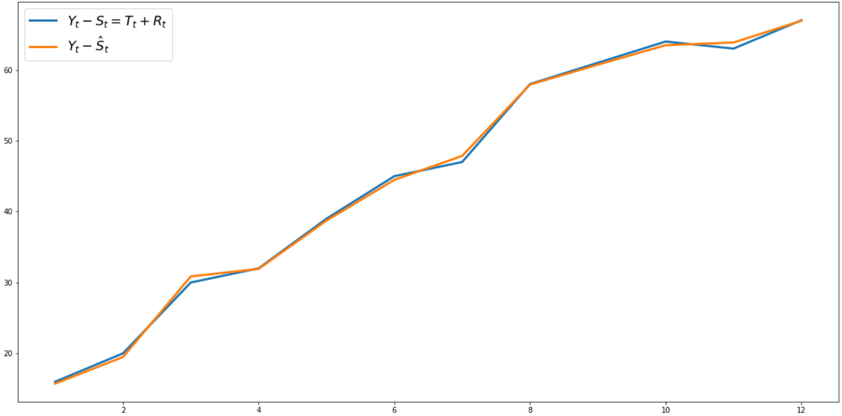
Παράδειγμα



Παράδειγμα



Παράδειγμα



$$[\alpha_{-s} \cdots \alpha_{-2}, \alpha_{-1}, \alpha_0, \alpha_1, \alpha_2, \ldots \alpha_s]$$

$s = 3$

$2s + 1$ Τάξη $\qquad \alpha_j = \frac{1}{2s+1}$

$2s$ Τάξη. $\qquad \alpha_j = \frac{1}{2s} \qquad \alpha_{-s} = \alpha_s = \frac{1}{4s}$

$$[Y_1, Y_2, Y_3, Y_4, Y_5, \ldots]$$

$\sqcup \quad \sqcup \quad \sqcup \quad Y_4$

$p = 2s + 1$

$p = 2s$

$s = p // 2$

Κατασκευή του φίλτρου.

$$[1, 1, \dots\dots, 1] \longrightarrow \left[\frac{1}{p}, \frac{1}{p}, \dots\dots, \frac{1}{p}\right]$$

$\downarrow$ εαν p άρτιος

$$[\cdot/2 \qquad\qquad \cdot/2]$$

υπόλοιπο

$p \% 2$

$\sqcup\ \sqcup\ \ldots\ \sqcup\ D_{s+1}, \ldots, D_{N-s}, \sqcup, \sqcup, \ldots, \sqcup$

$S_1, \ldots, S_p$ $\qquad p = 2s+1 \qquad p = 7$

$\sqcup \sqcup \sqcup, D_4, D_5, D_6, D_7, D_8, D_9, D_{10}, D_{11}, D_{12}, \sqcup, \sqcup, \sqcup$

$S_1\ S_2\ S_3\ S_4\ S_5\ S_6\ S_7\ S_1\ S_2\ S_3\ S_4\ S_5$

$$S_1 + R_8 \approx D_8$$

$$S_4 + \frac{R_4 + R_{11}}{2} \approx \frac{D_4 + D_{11}}{2}$$

$$S_t \quad [\underbrace{S_1, S_2, \ldots, S_p}, S_1, S_2, \quad]$$

$$\begin{array}{ccc} \begin{bmatrix} \hat{S}_1 & \hat{S}_2 & \hat{S}_p \end{bmatrix} \\ 0, & 1 & p-1 \end{array}$$

j από 0 μέχρι $N-1$

$$k = j \% p$$

νέο διάνυσμα$[j] = \hat{S}[k]$

$$\hat{S}_t, \forall t = 1, \ldots, N$$

[illegible] $= Y_t - \hat{S}_t \approx T_t + R_t$